10/02/2024

1. Solve $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - y = xe^{2x}$

Solution: For DF, $\left(\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - y\right) = xe^{2x}$

$$\Rightarrow \left(\frac{d^3}{dx^3} - 3\frac{d^2}{dx^2} + 3\frac{d}{dx} - 1\right)y = xe^{2x}$$

$$\Rightarrow (D^3 - 3D^2 + 3D - 1)y = xe^{2x}$$

For CF

$$D^3 - 3D^2 + 3D - 1 = 0$$

$$\Rightarrow (D-1)^3 = 0 \Rightarrow D = 1, 1, 1.$$

$$\therefore \; CF = (C_1 + C_2x + C_3x^2)e^x$$

For PI

$$PI = \frac{1}{(D-1)^3} xe^{2x}$$

very important

जब PI में $x^m e^{ax}$ type रहे तो

पहले e^{ax} को integrate करेंगे।

इसके लिए D के स्थान पर D+a रखेंगे

तथा e^{ax} को आगे लिखेंगे।

पुनः x^m को integrate करने के लिए

हर को Binomial theorem पर expand करेंगे।

$$\therefore PI = e^{2x} \frac{1}{[(D+2)-1]^3} x$$

$$= e^{2x} \cdot \frac{1}{(D+1)^3} x$$

$$= e^{2x} [1+D]^{-3} x$$

$$= e^{2x} [1 - 3D + \text{higher powers of } D] x$$

$$= e^{2x} [x - 3D(x) + \text{higher derivatives of } x]$$

$$= e^{2x} [x - 3 + 0] = (x-3)e^{2x}$$

$\therefore$ complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = (c_1 + c_2 x + c_3 x^2)e^{x} + (x-3)e^{2x}$$

2. Solve $(D^2 - 1)y = x^2 e^x$

$\Rightarrow$ For CF, $D^2 - 1 = 0 \Rightarrow D = \pm 1$

$$\therefore CF = c_1 e^{x} + c_2 e^{-x}$$

For PI

$$PI = \frac{1}{(D^2-1)} \cdot x^2 e^x$$

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$\boxed{\text{Impt. step}}$

$$\therefore PI = e^x \frac{1}{(D+1)^2 - 1} x^2$$

$$\Rightarrow PI = e^x \cdot \frac{1}{(D^2+2D+1-1)} x^2$$

$$= e^x \frac{1}{D^2+2D} x^2$$

$$= e^x \frac{1}{2D\left(1+\frac{D}{2}\right)} x^2$$

$$= e^x \frac{1}{2D}\left(1+\frac{D}{2}\right)^{-1} x^2$$

$$= \frac{1}{2} e^x \frac{1}{D}\left[1 - \frac{D}{2} + \left(\frac{D}{2}\right)^2 - \left(\frac{D}{2}\right)^3 + \cdots\right] x^2$$

Note $D(x^2) = 2x$, $D^2(x^2) = 2$ and $D^3(x^2) = 0$, Also other higher derivatives of $x^2 = 0$.

$$= \frac{1}{2} e^x \cdot \frac{1}{D}\left[x^2 - \frac{D(x^2)}{2} + \frac{D^2(x^2)}{4} - 0 + 0\right]$$

$$= \frac{1}{2} e^x \frac{1}{D}\left[x^2 - \frac{2x}{2} + \frac{2}{4}\right] = \frac{1}{2} e^x \frac{1}{D}\left(x^2 - x + \frac{1}{2}\right)$$

$$= \frac{1}{2} e^x \frac{1}{D}\left(x^2 - x + \frac{1}{2}\right) = \frac{1}{2} e^x \int \left(x^2 - x + \frac{1}{2}\right) dx$$

$$= \frac{1}{2} e^x \left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{2}\right) + K.$$

Hence Sol^n = CF + PI